Traceability on RSA-Based Partially Signature with Low Computation *

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Abstract

In this article, we show that the Chien et al's partially blind signature scheme based on RSA public cryptosystem could not meet the untraceability property of a blind signature.

Keywords: blind signature, electronic cash, untraceability.

1 Introduction

The concept of the blind signature was first introduced by Chaum [3]. It is an important technique to protect the right of an individual's privacy while one was shopping or voting over the Internet. Different from a regular digital signature scheme [6, 8, 9], the two additional required properties of a blind signature [7, 13] are as follows. *Blindness* means the signer of the blind signature does not see the content of the message and *untraceability* means the signer of the blind signature is unable to link the message-signature pair after the blind signature has been revealed to the public.

A blind signature also can be applied to electronic cash. To prevent double spending and reduce the size of the database of the electronic cash system [10, 11], partially blind signatures were proposed [1, 5]. In 2001, Chien et al. [4] proposed a partially blind signature scheme based on RSA cryptosystem [2, 12] that could reduce the computation load. However, in this article, we show that Chien et al's scheme failed to meet the untraceability property of a blind signature.

2 Chien et al.'s Partially Blind Signature Scheme

Recently, Chien et al. [4] proposed a partially blind signature scheme which is based on RSA public-key cryptosystem [12]. This scheme is divided into four phases: (1)*initialization*, (2)*requesting*, (3)*signing*, and (4)*extraction and verification* phases. The procedures of this scheme are listed as follows.

- **Initialization.** The signer chooses two distinct large primes p and q at random and computes $n = p \cdot q$. Let e be a public key such that $gcd(e, \phi(n)) = 1$, where $\phi(n) = (p-1) \cdot (q-1)$. And then calculate a privacy key d such that $e \cdot d = 1 \bmod \phi(n)$. The signer makes (e, n) as his/her public parameters and keeps (p, q, d) secretly.

- **Requesting.** The requester prepares the common information a, according to the predefined format, and the message m. The requester selects randomly two integers r and u in Z_n^* and then he/she computes $\alpha = r^e H(m)(u^2+1) \bmod n$, here $H(\cdot)$ denotes a one-way hash function. Finally, the requester sends the tuple (a, α) to the signer.

  After receiving (a, α), the signer verifies the common information a at first. And then the signer randomly chooses an integer x $(x < n)$ and sends it to the requester.

  After receiving x, the requester selects randomly an integer k and computes $b = r \cdot k$ and $\beta = b^e(u - x) \bmod n$. Then the requester sends β to the signer.

- **Signing.** Upon receiving β, the signer computes $\beta^{-1} \bmod n$ and $t = h(a)^d(\alpha(x^2+1)\beta^{-2})^{2d} \bmod n$ and then sends (β^{-1}, t) to the requester.

- **Extraction and verification.** After receiving (β^{-1}, t), the requester computes $c = (u \cdot x + 1)\beta^{-1} \cdot b^e \bmod n$ and $s = t \cdot r^2 \cdot k^4 \bmod n$. The tuple (a, c, s) is a digital signature on the message m. Any one can verify the signature (a, c, s) by checking if $s^e = H(a)H(m)^2(c^2+1)^2 \bmod n$.

The correctness of the above protocol is shown in [4].

3 The Weakness of Chien et al's Scheme

In this section, we show that Chien et al's partially blind signature scheme could not meet the untraceability property of a blind signature. The signer will keep a set of records for all blinded messages and use them to link a valid signature (a, c, s, m) to its previous signing process instance. The procedures of this cryptanalysis are listed as follows.

1. The signer can keep a set of records $\{\alpha, x, \beta, t, \beta^{-1}\}$, for all blinded messages.

2. When the requester reveals (a, c, s, m) to the public, the signer can link it using the kept records. Since $c = (u \cdot x + 1)\beta^{-1} \cdot b^e = (u \cdot x + 1)(u - x)^{-1} \bmod n$, the signer can derive a parameter $\acute{u}$ by computing $\acute{u} = (1 + c \cdot x)(c - x)^{-1} \bmod n$.

3. Since $\beta = b^e(u - x) \bmod n$, the signer can derive a parameter $\acute{b}$ by computing $\acute{b} = (\beta(\acute{u} - x)^{-1})^d \bmod n = \beta^d(\acute{u} - x)^e \bmod n$.

4. Since $\alpha = r^e H(m)(u^2 + 1) \bmod n$, the signer can derive a parameter $\acute{r}$ by computing $\acute{r} = \alpha^d H(m)^e(\acute{u}^2 + 1)^e \bmod n$.

5. Since $b = rk$, the signer can derive a parameter $\acute{k}$ by computing $\acute{k} = \acute{b} \cdot \acute{r}^{-1}$.

6. Finally, the signer can check if $s = t \cdot \acute{r}^2 \cdot \acute{k}^4 \bmod n$. If the result is true, the signer can link this signature.

From the above procedures, the partially blind signature of the requester can been trace.

4 Conclusion

In this article, we have shown that a cryptanalysis of Chien et al.'s partially blind signature scheme and the scheme could not meet the requirements of the untraceability property of a blind signature.

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