An Improvement of a Dynamic Cryptographic Key Assignment Scheme in a Tree Hierarchy *

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Abstract

This paper shows how several security classes in Liaw-Wang-Lei's cryptographic key assignment scheme can collaborate to derive the secret key of their immediate ancestor in some cases. We also propose two schemes which are a slight modification of the proposed scheme to enhance the level of security.

Keywords: Access control, cryptography, data security, multilevel.

1 Introduction

Recently, Liaw, Wang, and Lei [1] proposed an efficient cryptographic key assignment scheme for solving the access control problem in a tree structure. Basically, the scheme is based on Newton's interpolating method and a one-way function. The scheme not only achieve dynamic key assignment, but also are simple and efficient for generating and derivating keys. However, we show that several security classes can collaborate to derive the secret key of their immediate ancestor in this article. We also propose two schemes for modifying the Liaw-Wang-Lei's scheme slightly so that the security will be greatly improved.

2 The Weakness of Liaw-Wang-Lei's Scheme

Recently, Liaw, Wang, and Lei proposed an efficient dynamic cryptographic key assignment scheme, which was based on Newton's interpolating method and a one-way function, for solving the access control problem in a hierarchy.

In the subject paper [1], the authors assigned each security class C_i an associated distinct pair $(t1_i, t2_i)$ as the public parameter and a secret key K_i. Assume that the security class C_i has d immediate successors $C_{i_1}, C_{i_2}, \cdots, C_{i_d}$.

The security class C_i constructs an interpolating polynomial $NP_i(x)$ of degree d by interpolating on the points $(0, K_i)$, $(t1_{i_j}, t2_{i_j})$, $1 \leq j \leq d$, over $GF(P)$, where P denotes a large prime number. Let $NP(x) = (\alpha_d(x - x_{d-1})(x - x_{d-2}) \cdots (x - x_0) + \cdots + \alpha_1(x - x_0) + \alpha_0) \bmod P$, where $(\alpha_0, x_0) = (0, K_i)$, and $(\alpha_j, x_j) = (t1_{i_j}, t2_{i_j})$, $1 \leq j \leq d$. The secret key K_{i_j} of C_{i_j} is calculated by his immediate ancestor C_i, $K_{i_j} = F(NP_i(t2_{i_j})) \bmod P$, for $j = 1, 2, \cdots, d$, where F denotes a pseudo one way function, $F(X) = X^2 + 1 \bmod P$. The pairs of public parameters $(t1_{i_j}, t2_{i_j})$, P, and F are known to all security classes in the scheme. The security class C_i keeps only its own key K_i secret.

To derive K_{i_j} of C_{i_j}, the security class C_i reconstructs the interpolating polynomial $NP_i(x)$ by interpolating on the points $(0, K_i)$, $(t1_{i_1}, t2_{i_1})$, $(t1_{i_2}, t2_{i_2})$, $\cdots$, $(t1_{i_d}, t2_{i_d})$. The secret key K_{i_j} is thus obtained by computing $F(NP_i(t2_{i_j})) \bmod P$, where $t2_{i_j}$ is the public key of C_{i_j}.

The weakness in the security of Liaw-Wang-Lei's scheme is as follows. Let C_{i_j}, $1 \leq j \leq d$, be d immediate successors of the security class C_i. Since the points $(t1_{i_j}, t2_{i_j})$ for C_{i_j}, $j = 1, \cdots, d$, are known to each security class, we can construct an interpolating polynomial $NP_i(x)$ with one unknown point $(0, K_i)$ and d known points $(t1_{i_j}, t2_{i_j})$, $j = 1, \cdots, d$. The formula is as follows.

$$
\begin{aligned}
NP_i(x) &= (\alpha_d(x - x_{d-1})(x - x_{d-2}) \cdots (x - x_0) + \cdots \\
&\quad + \alpha_1(x - x_0) + \alpha_0) \bmod P, \\
&= e_1 + e_2 K_i \bmod P,
\end{aligned} \tag{1}
$$

where $(\alpha_0, x_0) = (0, K_i)$, $(\alpha_j, x_j) = (t1_{i_j}, t2_{i_j})$, $1 \leq j \leq d$, and e_1, e_2 are constants. Next, we take the polynomial $NP_i(t2_{i_j})$ in Equation (1) as a parameter of F. Since F is a one-way function of degree d, we have

$$
\begin{aligned}
F(NP_i(t2_{i_j})) &= K_{i_j}, \\
&= (e_{j_d} K_i^d + e_{j_{(d-1)}} K_i^{d-1} + \cdots + \\
&\quad \cdots + e_{j_1} K_i + e_{j_0}) \bmod P, \text{ for } j = 1, 2, \cdots, d.
\end{aligned}
$$

By Gauss's elimination method [2], there are a unique solution for d equations with d variables. Therefore, d immediate successors of security class C_i can collaborate to derive the secret key K_i of C_i. In fact, only one security class is needed to break the scheme when the one-way function is in Quadratic Residuosity modulo (i.e., $F(X) = e_2X^2 + e_1X + e_0 \bmod P$.) [3]. When the one-way function is a polynomial of degree d, only $d-1$ immediate successors are needed for a conspiracy attack. The following example illustrates this case.

Example 2.1

Suppose that there are nine security classes in the system, as shown in Figure 1. Let the prime number $P = 13$ and the one-way function $F(X) = X^2 + 1 \bmod 13$. Under the proposed scheme, the secret key K_i and public parameters $(t1_{i_j}, t2_{i_j})$ for each security class C_{i_j} are as shown in Table 1. The following steps will show how C_4 and C_5 can collaborate to derive the secret key of C_1.

1. Construct an interpolating polynomial of C_1 with one unknown variable K_1. So we have

$$NP_1(x) = 2(x-8)(x-K_1) + (x-K_1) \bmod 13. \tag{2}$$

2. Substitute the public parameters ($t2_{14}$ and $t2_{15}$) of C_4 and C_5 into Equation (2), respectively.

$$\begin{aligned} NP_1(t2_{14}) &= NP_1(8) = 8 - K_1 \bmod 13, \\ NP_1(t2_{15}) &= NP_1(3) = 12 + 9K_1 \bmod 13. \end{aligned} \tag{3}$$

3. Substitute $NP_1(t2_{14})$ and $NP_1(t2_{15})$ into the one-way function ($F(X) = X^2 + 1 \bmod 13$) with the secret keys (K_4 and K_5) of C_4 and C_5, respectively. Thus,

$$11 = (8 - K_1)^2 + 1 \bmod 13,$$

$$= \quad (K_1^2 + 10K_1) \bmod 13, \tag{4}$$

and

$$\begin{aligned} 4 &= (12 + 9K_1)^2 + 1 \bmod 13, \\ &= (3K_1^2 + 8K_1 + 2) \bmod 13. \end{aligned} \tag{5}$$

4. By the Gaussian elimination method, we obtain $K_1 = 2$ from Equation (4) and Equation (5).

In fact, either C_4 or C_5 can derive the secret key of C_1 because the one-way function is Quadratic Residuosity modulo [3].

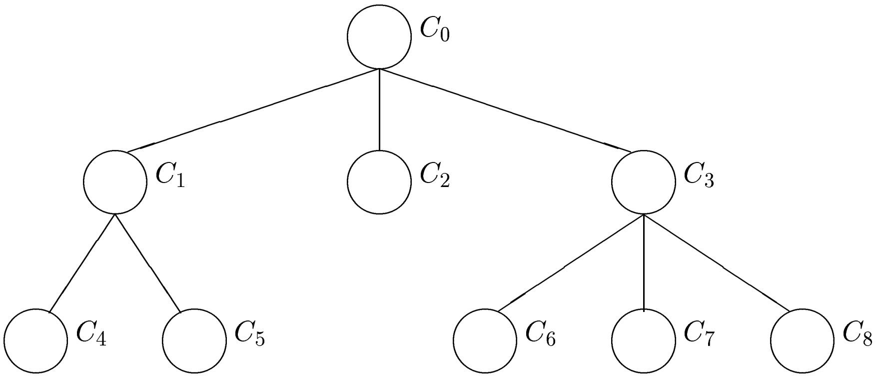

Figure 1: An example of the tree structure.

Table 1: The keys and public parameters for each security class.

	Public key pair $(t1_i, t2_i) = (\alpha_i, x_i)$	Secret key $K_i = F(NP(t2_i))$
C_0		3
C_1	$(1, 4)$	2
C_2	$(2, 7)$	5
C_3	$(3, 6)$	10
C_4	$(1, 8)$	11
C_5	$(2, 3)$	4
C_6	$(1, 6)$	4
C_7	$(2, 9)$	11
C_8	$(3, 8)$	5

3 Our Schemes

We have shown that Liaw-Wang-Lei's scheme is insecure. Since the purpose of the Newton's interpolate polynomial is only hiding secret keys in their scheme, the powerful and high secure one-way function is needed.

From the above discussion and example, we see that $d-1$ security classes can collaborate to attack the one-way function of degree d. We now give two extended schemes which improve of Liaw-Wang-Lei's scheme for withstanding this type of attack.

Scheme 1: Take different prime number P within Galois field $GF(P)$ to the Newton's interpolating polynomial and the predefined one-way function. For example, Let P_1 and P_2 be two large but not equal prime numbers. We can construct $NP(x) = (\alpha_d(x - x_{d-1})(x - x_{d-2}) \cdots (x - x_0) + \cdots + \alpha_1(x - x_0) + \alpha_0) \bmod P_1$ as Newton's interpolating polynomial for each security class in the system. And take $F(X) = X^2 + 1 \bmod P_2$ as our predefined one-way function.

Scheme 2: Choose a one-way function of degree $d+2$, where d is the maximal number of immediate successors of each security class in the whole system.

The above extended schemes not only retains the advantages of the scheme in [1] but also enhances the security.

4 Conclusions

We have shown how several security classes in Liaw-Wang-Lei's cryptographic key assignment scheme can collaborate to derive the secret key of their immediate ancestor in some cases. We also have proposed two extended schemes which are a slight modification of the proposed scheme. The proposed schemes not only retains the advantages of Liaw-Wang-Lei's scheme but also enhances the security.

References

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